IEooc_Application1_Exercise1: The end of area? Sociometabolic regimes and their area demand

**Goal:** Develop a simple engineering model, learn about the physical distance and population constraints in the agricultural society, and estimate the area yield of modern renewable energy technologies

**Part 1, agricultural societies** Biomass (wood or grain) is the main source of energy in agricultural societies. On land, biomass also serves as transportation fuel. Bairoch (1993) estimates that in the 18th century, the transport costs of one tonne for 1 km on land were equivalent to 3.9 kg grain. Area yields for grain in France in the 18th century were about 50 tonnes per km² (Sieferle et al., 2006). Imagine a large country where all land is used to farm grain. 80% of the grain is used to feed the rural population, and the remaining 20% are transported to the capital using horse carriages, and the draft horses are fed from the grain they transport, consuming 3.9 kg grain for each tonne-km of transport.

**Questions:**

1) What is the functional relationship between the original mass of grain loaded on the carriage and the mass arrived at the city after a transport distance \( r \)?

For each distance \( \Delta x \) a mass \( m \) is transported the transport costs in form of mass are \( a \cdot \Delta x \), where \( a = 0.0039 \) t per t·km. These costs are taken from the original mass (the horses eat up part of their load), so that we have a mass decline

\[
\Delta m = - a \cdot \Delta x \cdot m = - a \cdot m \cdot \Delta x
\]

This relation between mass and mass change leads to a differential equation

\[
\frac{\Delta m}{\Delta x} = - a \cdot m
\]

Whose solution is the exponential decline function, where \( m_0 \) is the original mass present at the start (https://en.wikipedia.org/wiki/Exponential_decay).

\[
m(x) = m_0 \cdot e^{-ax}
\]

As the horses eat up their load the load they transport becomes smaller, and the mass-distance-relationship is an exponential decay. The rest energy that the draft horses have to eat when not pulling any load is not considered here.
2) Imagine that grain from all provinces is transported to the capital using a radial road network with the capital at the centre. How much grain arrives at the city if the country is a) circular with a radius of 400 km, and b) very, very large? This formula might be of help:

\[
\int_{0}^{R} r \cdot e^{-a \cdot r} \, dr = \frac{1}{a^2} \left( \frac{R}{a} + \frac{1}{a^2} \right) \cdot e^{-a \cdot R}
\]

With the data in the text, the area density of the grain available for transport into the city is \( b = 10 \text{ t/km}^2 \). That number comes from the area yield of 50 t/km² and the share of 20% that is available for transport. With the result of question 1, we now the mass that arrives from distance \( r \) from the city. Imagine a ring around the city in distance \( r \) and width \( \Delta r \). The mass of the grain that is available from that ring is \( \Delta m = 2 \cdot \pi \cdot r \cdot \Delta r \cdot b \), and the amount that arrives at the city is

\[
\Delta m(r) = 2\pi \cdot r \cdot \Delta r \cdot b \cdot e^{-a \cdot r}
\]

The total mass arriving at the city from a circle shape region starting at distance 0 and extending until distance \( R \) is

\[
M(R) = 2\pi \cdot b \cdot \int_{0}^{R} r \cdot e^{-a \cdot r} \, dr = 2\pi \cdot b \cdot \left[ \frac{1}{a^2} \left( \frac{R}{a} + \frac{1}{a^2} \right) \cdot e^{-a \cdot R} \right]
\]

Here, we used the integral from above.

We calculate the total mass arriving for the two cases:

a) \( R = 400 \text{ km} \): \( M = 1.91 \text{ Mt Grain} \).

b) \( R = \infty \): \( M = 4.13 \text{ Mt Grain} \).

It is instructive to see the grain-distance relationship to understand the behaviour of the model:
3) With an energy content of the grain of 15 GJ/ton and a typical energy need of a agricultural citizen of 50 GJ/cap/yr (Haberl et al., 2011), what would be the maximum population of the city for the cases a and b? Assume that grain is the only energy source this country has.

Population = Mass of grain * energy per mass of grain / energy per person.

   a) R = 400 km: P = 570 000.
   b) R = ∞: P = 1.2 million.

4) Are these values realistic or do they represent under- or overestimates and why?
On the one hand these numbers are overestimates because not all land can be used for farming and the draft animals also need to eat even when they do not transport anything. For example, the area needs of the road network were not accounted for. On the other hand, more energy-efficient modes of transport, especially transport on canals, can reduce costs by a factor of up to ten (Sieferle et al., 2006).
Part 2, present and future societies) During the industrial revolution the area constraint set by the agricultural yields and transport energy costs was overcome. With the coming energy technology revolution a massive switch back to area-constrained technologies is bound to happen. A set of different renewable energy technologies is available. Imagine a circle-shaped country or region with a diameter of 200 km. Assume that the entire area can be covered with renewable energy installations.

Questions:

1) How much electricity can be supplied by that area with the three primary energy carries sunlight, wind, and biomass? (Rough values), use area yields by MacKay (2008) or other sources. With MacKay’s yield factors for 1 m$^2$ of land:
   - Solar: 20% efficiency of an average daily irradiance of 110 W/m$^2$ (solar irradiance: 1 kW/m$^2$, for 1000 sunshine hours and 8760 hours per year): 200 kWh/ m$^2$·yr.
   - Wind: 2W/m$^2$ are provided, that is about 17 kWh/ m$^2$·yr.
   - Biomass: Best plants are 2% efficient in converting sunlight to energy stored in biomass, but 0.5% is more common (Fig. 6.11): 5 kWh/ m$^2$·yr.

2) How many people can be supplied with electricity, given that the typical energy need of a citizen in an industrial society is around 300 GJ/cap/yr (Haberl et al., 2011) and we assume that all energy is consumed in form of electricity (1 kWh is 3.6 MJ)?
   - Total area: 3.14e10 m$^2$. Total energy: 300 GJ are 83333 kWh.
   - Number of people:

   Solar: 75 million  Wind: 6.5 million  Biomass: 2 million

3) Are these values realistic or do they represent under- or overestimates and why?

These values are overestimates. Not all area can be converted to farm energy. The intermittency issue was not addressed here either. On a global scale a circle with a radius of 200 km is not a large area. We would (with appropriate storage) need about 100 of these circles to deliver to 7.5 billion people the energy consumed on average in the richest countries.

The big difference between solar energy density and biomass energy density is striking.

References: