Part II: Methods

Methodology 3: Dynamic Material Flow Analysis
IEooc_Methods3_Lecture3

Inflow-driven and stock-driven modelling

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Content

The inflow-driven model: Determining the stock from historic inflows

The stock-driven model: Determining inflows from a stock scenario
The inflow-driven model: Research questions

In-use stocks of buildings, vehicles, and infrastructure provide services to people and are a central determinant of sustainable development.

For many stocks, in particular, material stocks estimates are not available.

Material stocks can be determined
a) ‘bottom-up’: from building and vehicle statistics and product-specific material content data
b) ‘top-down’: from aggregated consumption data and a lifetime model

Applying the inflow-driven model to estimate in-use stocks (discrete time with interval $\Delta t$, typ. 1 year)

System definition:

1) Determine apparent consumption: $C(t) = P(t) + I(t) - E(t)$

2) Apply the lifetime model: $O(t) = \sum_{\tau=t_0}^{t} C(\tau) \cdot LTD(t - \tau)$

3) Determine stock change $\Delta S(t) = C(t) - O(t)$

4) Determine stock $S(t) = S(t_0) + \sum_{t_0}^{t} \Delta S(\tau)$

$\Delta S(t)$: discrete time change

$dS(t)$: continuous time change

$LTD(x)$: discrete lifetime distribution, unit is 1, meaning is probability of discard at age $x$. 
Applying the inflow-driven model to estimate in-use stocks (continuous time)

System definition:

1) Determine apparent consumption: \[ C = P + I - E \]

2) Apply the convolution: \[ O(t) = \int_{t_0}^{t} C(\tau) \cdot pdf(t - \tau) \, d\tau \]

3) Determine stock change \[ \Delta S(t) = C(t) - O(t) \]

4) Determine stock \[ S(t) = S(t_0) + \int_{t_0}^{t} dS(\tau) \, d\tau \]
Product lifetimes in the inflow-driven model

Lifetime data are obtained or have to be inferred from surveys, statistical records, and anecdotal evidence (newspaper report, blog entries, Wikipedia).

Product lifetimes in the inflow-driven model

Lifetime data are scattered across the literature and often show large variations:


Normal, Gamma, Exponential, and Weibull distributions are commonly applied to model product lifetimes.
Implementing the inflow-driven model

**In Excel:** The pdf can be generated using NORM.DIST, the rest is a standard Excel computation.

**In Python:** via the dynamic_stock_model class: [https://github.com/stefanpauliuk/dynamic_stock_model](https://github.com/stefanpauliuk/dynamic_stock_model)

```python
TestDSM = DynamicStockModel(t = np.arange(1,11,1), i = np.arange(2.5,12.5,1), lt = {'Type': 'Normal', 'Mean': np.array([5]), 'StdDev': np.array([1.5]) })

Stock_by_cohort, ExitFlag = TestDSM.compute_s_c_inflow_driven()

O_C, ExitFlag = TestDSM.compute_o_c_from_s_c()
```

→ Applied in IEooc_Methods3_Exercise1 (Excel) and IEooc_Method3_Software1 (Python)
The inflow-driven model

Apparent saturation of per capita steel stocks in some industrialized countries

Source: Müller et al. (2011): DOI 10.1021/es102273t
The inflow-driven model


Source: Pauliuk et al. (2012), DOI 10.1021/es201904c
The stock-driven model for determining inflows

To build scenarios for the future development of material cycles, one can
• Extrapolate or assume future consumption levels and then calculate the stocks and the services provided.
• Extrapolate or assume future service levels, infer the stocks required to deliver them, and calculate the inflows required to expand and maintain those stocks.

The latter approach is often more realistic, as it allows us to link the actual outcome of economic activity, service provision, directly to the socioeconomic variables providing those services, stocks.

The research question is then:

*How large is the inflow needed to maintain and expand the in-use stock so that it fits a given scenario?*

The method that answers this question is called stock-driven modelling.

Mathematically, the determination of an inflow from an outflow is the inverse of the convolution operation used for the inflow-driven model.

As the convolution involves the calculation of average values, its application to functions in general leads to a loss of information.

That means that the original signal (or inflow) can be reconstructed from the filtered signal (the outflow or stock) only in special cases.

In dynamic MFA such a special case is given if the age-cohort composition of the initial stock is known or that stock is zero.
Procedure of the stock-driven model to estimate in-use stocks

System definition: \( I \xrightarrow{\text{Process}} O \) (discrete time \( \Delta t \), e.g., 1 yr)

Recursive procedure: Starting in the first model year, repeat the following steps for each year:

1) Calculate the outflow from the existing stock using the convolution of historic inflows:

\[
O(t) = \sum_{\tau=t0}^{t} I(\tau) \cdot LTD(t - \tau)
\]

2) Calculate the gap \( \Delta S(t) \) between the actual stock and the remaining stock after \( O(t) \) removed:

\[
\Delta S(t) = S_{\text{ext}}(t) - S(t) = S_{\text{ext}}(t) - \sum_{i0}^{t} \left( I(\tau) - O(\tau) \right) = S_{\text{ext}}(t) - \sum_{i0}^{t} I(\tau) - \sum_{i0}^{t} \sum_{i=0}^{t} I(i') \cdot LTD(t' - t'' = t0)
\]

3) Set the inflow to fill the gap \( \Delta S(t) \):

\[
I(t) = \Delta S(t)
\]

Then repeat from step 1.

Here, \( LTD(x) \) is the discrete lifetime distribution, unit is 1, meaning is probability of discard at age \( x \).
Practical exercise: a simple dynamic stock model for the vehicle fleet

stock-driven model, fixed product lifetime: 3 years

<table>
<thead>
<tr>
<th>Year</th>
<th>Stock (cars)</th>
<th>Age-cohort</th>
<th>Use (km/yr)</th>
<th>Emissions (CO₂/yr)</th>
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</thead>
<tbody>
<tr>
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**Efficiency (CO₂/km)**

6 5 5 4 4 3
Practical exercise: a simple dynamic stock model for the vehicle fleet

stock-driven model, fixed product lifetime: 3 years

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</table>

Efficiency (CO\(_2\)/km):

|          | 6 | 5 | 5 | 4 | 4 | 3 |

Calculation:

\[
240 \text{ CO}_2/\text{yr} = 4 \text{ cars} \times 10 \text{ km/yr} \times 6 \text{ CO}_2/\text{km} \\
261 \text{ CO}_2/\text{yr} = 4 \text{ cars} \times 9 \text{ km/yr} \times 6 \text{ CO}_2/\text{km} + 1 \text{ car} \times 9 \text{ km/yr} \times 5 \text{ CO}_2/\text{km}
\]
Practical exercise: a simple dynamic stock model for the vehicle fleet

stock-driven model, fixed product lifetime: 3 years

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**Efficiency (CO₂/km)**

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**Calculations**

240 CO₂/yr = 4 cars * 10 km/yr * 6 CO₂/km

261 CO₂/yr = 4 cars * 9 km/yr * 6 CO₂/km + 1 car * 9 km/yr * 5 CO₂/km
Practical exercise: a simple dynamic stock model for the vehicle fleet

stock-driven model, fixed product lifetime: 3 years

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Efficiency (CO$_2$/km):

6 5 5 4 4 3

240 CO$_2$/yr = 4 cars * 10 km/yr * 6 CO$_2$/km

261 CO$_2$/yr = 4 cars * 9 km/yr * 6 CO$_2$/km
+ 1 car * 9 km/yr * 5 CO$_2$/km
# Practical exercise: a simple dynamic stock model for the vehicle fleet

- **stock-driven model, fixed product lifetime: 3 years**

## Age-cohort

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## Use (km/yr)

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## Emissions (CO₂/yr)

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240 CO₂/yr = 4 cars * 10 km/yr * 6 CO₂/km

261 CO₂/yr = 4 cars * 9 km/yr * 6 CO₂/km + 1 car * 9 km/yr * 5 CO₂/km

## Efficiency (CO₂/km)

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The stock-driven model for determining inflows

*The stock-driven model is the inverse of the inflow driven model:*

- The inflow computed by the stock-driven model is identical to the original inflow.
- The stock computed with the inflow-driven model is identical to the original stock.

*Be creative with the initial stock!*

- Use stock obtained from inflow-driven model to apply stock-driven model from a time when there were virtually no stocks.
- If the original stock age-cohort composition is unknown, the leaching model can be applied to $S_0$. 
Implementing the stock-driven model

**In Excel:** Possible but cumbersome. Better use programming environment.

**In Python:** via the `dynamic_stock_model` class: [https://github.com/stefanpauliuk/dynamic_stock_model](https://github.com/stefanpauliuk/dynamic_stock_model)

```python
TestDSMX = DynamicStockModel(t = np.arange(1,11,1),
s = np.array([ 2.5, 6. , 10.5, 16. , 22.5, 27.5, 32.5, 37.5, 42.5, 47.5]),
l = {'Type': 'Normal', 'Mean': np.array([4]), 'StdDev': np.array([1.0])})
S_C, O_C, I, ExitFlag = TestDSMX.compute_stock_driven_model
O, ExitFlag = TestDSMX.compute_outflow_total()
DS, ExitFlag = TestDSMX.compute_stock_change()
Bal, ExitFlag = TestDSMX.check_stock_balance()
```

→ Applied in IEooc_Method3_Software1 (Python)
The stock-driven model for determining inflows

Example 1: The future housing stock in the Netherlands

![Graphs showing population, lifetime distribution of dwellings, concrete use per UFA, UFA per dwelling, persons per dwelling, and UFA per capita over the years.]

Fig. 4 – Estimation of parameters functions for variants low (light grey), medium (dark grey), and high (black).

The stock-driven model for determining inflows

Fig. 6 – Simulation result for the medium variant.

The stock-driven model for determining inflows

Example 2: The future of the Chinese steel cycle

Source: Pauliuk et al. (2012): The Role of Stocks in the Chinese Steel Cycle. DOI: 10.1021/es201904c
Forecast of steel demand by region, 2005-2050 (a).

From a stock scenario, which was obtained from applying S-shaped saturation curves for the stock the final steel consumption (figure) and scrap generation by region and product group can be determined.
Example 4: Historic development of in-use stocks provides benchmarks for personal wealth in developing countries

Source: Pauliuk et al. (2012): . DOI 10.1021/es201799k
Example 4 ctd.

a) Emission reduction scenarios for passenger cars in China

- Baseline
- F - Fuel consumption: 6 → 4 / 5 → 2 (l/100 km)
- T - Low consumption car share: 10% → 33%
- L - Kilometrage: 15000 → 12000 km/yr
- C - Cars per 1000: 450 → 300
- P - Population: 1.4 → 1.25 billion
- Bottom Line

Global mean temperature increase about pre-industrial level (°C)

Stabilisation classes for 2050

- VI
- V
- IV
- III
- II
- I

Direct CO₂ emissions in Gt/yr

(1) Fleet average fuel consumption F (l/100km)